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A perfectly competitive economy is an economy without welfare relevant endogenous learning

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Abstract

The two fundamental theorems of welfare economics hold a central position in economic analysis and teaching. I demonstrate that perfect competition, a standard assumption underlying these theorems, preclude heterogeneous welfare relevant learning decisions. The reason is straightforward: if the learnt information is not shared with everyone, there is asymmetric information; if the information is shared, externalities arise. Thus, the standard conditions for the two welfare theorems are incompatible with the feasibility of endogenous welfare relevant learning, unless restrictions are imposed ensuring that consumers' learning choices are always identical.

Keywords: Perfect competition; fundamental welfare theorems; learning; symmetric information; externalities.

JEL codes: D41, D50, D60, D61, D62, D82.

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1. Introduction

The concept of a perfectly competitive economy occupies a central position in economic analysis and teaching, constituting the basis of the two fundamental theorems of welfare economics (Arrow 1951; Debreu 1959).

The first fundamental theorem, establishing the Pareto efficiency of the market equilibrium, requires only that the economy is perfectly competitive (Walrasian) and that preferences are locally non-satiated; the second theorem requires a few additional assumptions.[2] While the requirement of local non-satiation may be described as 'mild' (Hammond 1998), it is well-known that the conditions for an economy to be perfectly competitive are in themselves very demanding – precluding, for example, externalities and asymmetric information.

In the present note, I point out a logically simple but apparently often overlooked implication of these conditions: an economy in which consumers can choose to learn is generally not perfectly competitive. In fact, if welfare relevant endogenous learning is feasible, the standard conditions for perfect competition cannot hold except in the presence of restrictions guaranteeing identical learning choices by all consumers.

Grossman and Stiglitz (1976, 1980) point out that informationally efficient markets may be impossible: "[B]ecause information is costly, prices cannot perfectly reflect the information which is available, since if it did, those who spent resources to obtain it would receive no compensation" (Grossman and Stiglitz 1980, p. 405). The present argument, although related, is simpler. The main intuition is straightforward: if a consumer chooses to learn something, and the new information is not immediately shared with everyone else, asymmetric information results. If the new information is shared with everyone, however, another problem arises: symmetric information essentially makes welfare relevant knowledge a public good. Choosing to acquire such information, thus, in effect constitutes a private contribution to a public good, causing externalities. The resulting free-rider problem emerges irrespective of whether the information is dispersed directly or through markets.

Even in imperfectly competitive economies, Pareto efficiency may prevail in special cases.[3] My claim is not that endogenous welfare relevant learning necessarily causes the market

[2] See, e.g., Mas-Colell et al. 1995, Ch.16.

[3] For example, a polluting monopolist can happen to produce a socially efficient product quantity if the market failures of negative externalities and market power just outweigh each other.

equilibrium to be inefficient. In a perfectly competitive economy, however, a Pareto optimal market equilibrium is not merely possible – it is guaranteed. The analysis below provides a reminder of the extreme restrictiveness of the assumptions needed for such a guarantee to hold. Once heterogeneous, welfare relevant information acquisition choices are allowed, conflicts between the various conditions for perfect competition cannot be avoided.[4]

Below, I start by outlining the formal framework, before introducing some helpful definitions and concepts. I then turn to the most general mechanism by which chosen learning tends to involve market failure: it affects the economically relevant set of contingent commodities. I proceed to show how other welfare relevant aspects of learning – in particular, consumers having preferences over learning, and learning affecting technologies – cause market failure in broadly similar ways. I summarize these arguments by means of an impossibility result. Finally, I discuss certain interpretative issues – including why the problem is solved neither by the Coase theorem nor information dispersion through markets – before concluding.

2. The basic framework

The framework presented below builds on Mas-Colell et al. (1995, Ch. 16&19). Consider an economy with $I > 0$ consumers, $J > 0$ firms, and $L > 0$ commodities, while $S > 1$ is the set of possible states of the world.[5]

Each consumer $i = 1, \ldots, I$ is characterized by a consumption set $X_i \subset \mathbb{R}^{LS}$ and a complete and transitive preference $\succsim_i$ defined on X_i. Each firm $j = 1, \ldots, J$ is characterized by a nonempty and closed production technology $Y_j \subset \mathbb{R}^{LS}$. Initial resources are given by $\overline{\omega} = (\overline{\omega}_1, \ldots, \overline{\omega}_L) \in \mathbb{R}^{LS}$. Each consumer has an initial endowment vector $\omega_i \in \mathbb{R}^{LS}$, and gets a share $\theta_{ij} \in [0,1]$ of each firm j's profits. Contingent markets open before the state $s \in S$ is realized.

An allocation $(x, y) = (x_1, \ldots, x_I, y_1, \ldots, y_J)$ is a consumption vector $x_i \in X_i$ for each consumer i and a production vector $y_j \in Y_j$ for each firm j. Firms maximize profits, given prices and production technologies; consumers choose their maximally preferred bundles of (contingent) commodities given prices, budget sets and preferences.

[4] Patents clearly involve market power and are, for this reason, not discussed further below (see Hall and Harhoff 2012 for a survey).

[5] For convenience, I follow Mas-Colell et al. (1995, p. 688): "For simplicity we take S to be a finite set with (abusing notation slightly) S elements."

The above economy is *perfectly competitive* if i) there is universal price quoting of commodities (market completeness); and ii) all agents are price takers; and iii) information is symmetric: for every $i, m = 1, \ldots, I$, any two states $s, s' \in S$ are distinguishable by one consumer i if and only if these states are distinguishable by every other consumer m; and iv) there are no externalities.

Requirement iv) requires some further clarification. Mas-Colell et al. (1995, p. 352) specify externalities as being present if the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy. This definition is somewhat unsatisfactory for the present purpose: first, it may not be entirely clear how to interpret the word "well-being", since the description of preferences above is purely ordinal; second, as I will return to, the exact interpretation of the word "directly" matters. Below, I will take externalities to be present if, for some consumer i, the consumption set X_i, initial endowments ω_i, preferences $\succsim_i$, and/or profit shares θ_{ij} are affected by an action taken by another consumer $m \neq i$, or by a firm j such that $\theta_{ij} \neq 1$; externalities are also taken to be present if, for some firm j, the production technology Y_j is affected by an action taken by another firm $k \neq j$, or by a consumer i for whom $\theta_{ij} \neq 1$.

The market equilibrium of a perfectly competitive economy (an Arrow-Debreu equilibrium, Arrow and Debreu 1954) is an allocation (x^*, y^*) and a system of prices $p = (p_{1S}, \ldots, p_{LS}) \in \mathbb{R}^{LS}$ such that (a) for every j, y^* maximizes profits, i.e., $py \leq py^*$ for all $y_j \in Y_j$, and (b) for every i, x^* is the maximal of $\succsim_i$ in the budget set $\{x \in X: px \geq p\omega_i + \sum_j \theta_{ij}\, py^*\}$, and c) $\sum_i x_i^* = \bar{\omega} + \sum_j y_j$.[6]

The first fundamental theorem of welfare economics can now be specified as follows: If preferences are locally non-satiated, the market equilibrium of a perfectly competitive economy is Pareto optimal.[7]

3. Knowledge and learning

For simplicity, I assume that only consumers, not firms, can have knowledge (although firm owners can of course have knowledge).

[6] Mas-Colell et al. 1995, pp. 547-548 and 692.
[7] See Mas-Colell et al. 1995, p. 549.

Let consumer i's knowledge K_i be defined as the set of pairs of states $s, s' \in S$ such that s and s' are distinguishable by i.

More precisely, let Π_i be a partition of S into pairwise disjoint information sets $E_i \subset S$, such that any two states $s, s' \in S$ are indistinguishable by i if and only if there exists $E_i \in \Pi_i$ such that $s, s' \in E_i$ (Hammond 2005). K_i is then the set of pairs of states $s, s' \in S$ such that there does not exist $E_i \in \Pi_i$ for which $s, s' \in E_i$.[8] Furthermore, let k_i be the number of information sets (elements) in Π_i: the higher k_i, the finer is i's information partition Π_i.

Let K_i^0 be consumer i's initial knowledge, and let initial knowledge be symmetric (otherwise the economy obviously cannot be perfectly competitive). That is, $K_i^0 = K_m^0 = K^0$ for all $i, m = 1, \dots, I$.[9]

In what follows, it will be convenient to distinguish between active and passive learning. Assume that K^0 does not allow consumers to distinguish between two states s' and s'', but that a consumer may possibly observe an information signal $\varphi^{s',s''}$ enabling her to distinguish between these two states. Let *passive learning* mean that i's observation of the signal $\varphi^{s',s''}$ is not chosen by i, but imposed exogenously on her by nature or as a consequence of another agent's choices. Let Λ_i denote the additional set of pairs of states that i is able to distinguish as a result of i's *passive* learning (that is, in addition to the states included in K^0). Further, let *active learning* mean that i's observation of the signal $\varphi^{s',s''}$ is chosen by i, and let Φ_i denote the additional set of pairs of states that i is able to distinguish as a result of i's *active* learning. Thus, Φ_i is a choice variable, while Λ_i is not. After learning, consumer i's knowledge is given by $K_i = K^0 \cup \Lambda_i \cup \Phi_i$.

Below, I assume that active learning is feasible. To avoid complicating matters unnecessarily, I disregard passive learning imposed exogenously by nature unless otherwise explicitly noted.

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[8] This definition of knowledge allows me to treat learning simply as an expansion of knowledge. Alternatively, definitions could be based on σ-algebras.

[9] Note that even if information is symmetric at the time contracts are entered, asymmetric information caused by subsequent learning may compromise efficiency due to moral hazard problems. Consider, for example, the following possible timeline. In period 0, in which information is symmetric, all behavioral decisions are made and all contracts entered. In period 1, the last period, consumers observe the realized state, depending on their previous learning choices, and contingent contracts are carried through. Information may thus be asymmetric only in period 1, in which no more trade nor renegotiation of previous contracts take place. If, however, consumers foresee in period 0 that they may be unequally able to verify contract contingencies in period 1, they may abstain from entering otherwise Pareto improving contracts in period 0.

Symmetric information is preserved if there is never any learning, i.e., $\Lambda_i = \Phi_i = \emptyset$ for every $i = 1, \ldots, I$, ensuring that $K_i = K_m = K^0$, implying that the observable world is deterministic. Symmetric information is also preserved if any signal observed by consumer i is also observed by every other consumer m, that is, $\Lambda_i \cup \Phi_i = \Lambda_m \cup \Phi_m$ for all $i, m = 1, \ldots, I$, such that $K_i = K^0 \cup \Lambda_i \cup \Phi_i = K^0 \cup \Lambda_m \cup \Phi_m = K_m$ for all $i, m = 1, \ldots, I$.

Let *no spillovers* mean that for all $i, m = 1, \ldots, I$ such that $m \neq i$, consumer i's knowledge K_i is unaffected by consumer m's learning $\Lambda_m \cup \Phi_m$; hence, learning by one consumer (whether active or passive) does not cause passive learning by others. Let *complete spillovers* mean that any signal observed by a consumer i is immediately passively observed by every other consumer m, so that $\Lambda_i \cup \Phi_i = \Lambda_m \cup \Phi_m$ always for all $i, m = 1, \ldots, I$. Let *partial spillovers* mean that a subset of consumer i's learning is passively observed by some but not necessarily all other consumers $m \neq i$.

The fundamental reason why symmetric information is required in a perfectly competitive economy is the need for equal ability to verify contract contingencies. Learning can affect economic outcomes for two basic reasons: first, the learner might achieve an information advantage vis-a-vis others that might possibly be exploited to reap economic gains at those others' expense; second, the new information can be directly useful per se, for example by improving the learner's ability to stay healthy or by increasing productivity in a firm the learner owns. Let *welfare irrelevant* learning mean acquisition of information that would have no economically relevant consequences if the information were equally available to all. *Welfare relevant* learning is any other learning, that is, the acquisition of information that can affect economically relevant variables even in the case where the information were equally available to all.[10] More formally, i's learning $\Lambda_i \cup \Phi_i$ is welfare relevant if, for some $i, m = 1, \ldots, I$ (where possibly, $m = i$) and/or some $j = 1, \ldots, J$, $\Lambda_i \cup \Phi_i$ could feasibly affect X_m and/or $\succsim_m$ and/or Y_j and/or ω_m and/or θ_{mj} even if the information were symmetrically dispersed to all, i.e even if $K_i = K_m = K^0 \cup \Lambda_i \cup \Phi_i = K^0 \cup \Lambda_m \cup \Phi_m = K^0 \Lambda \cup \Phi$ for all $i, m = 1, \ldots, I$.

[10] Note that *welfare relevance* as defined here differs from *payoff relevance* as the latter term is used by Tirole (1982) and Milgrom and Stokey (1982).

4. Learning and market failure: affecting the relevant set of contingent commodities

If the initial knowledge K^0 were sufficient to allow consumers to distinguish between every state in S, there would be nothing left to learn. Assuming that learning is feasible thus amounts to assuming that initial information is imperfect, i.e., $k_i < S$.

Imperfect information does not necessarily imply missing asset markets, since consumers may choose to learn. It does, however, limit the set of distinguishable contracts. As pointed out by Mas-Colell et al. (1995, p.709), "a promise to deliver one unit of good is worthless if delivery cannot be enforced". There is little point in paying someone to send umbrellas conditional on the weather being rainy at time t unless you will be able to know whether it rains at time t. If consumer i cannot distinguish a state s' from another state s'', any contract on delivery of a commodity contingent on state s' is, from i's point of view, equivalent to the same contract contingent on state s''.

This limits the set of economically relevant contingent commodities. In particular, the consumption set X_i over which i's preferences is defined is essentially restricted by i's knowledge.

If $k_i < S$, some contingent commodities in S are not distinguishable from each other by i. From i's perspective, there can be no more than Lk_i distinguishable elements in X_i. That is, as judged by i, $X_i \subseteq \mathbb{R}^{Lk_i} \subseteq \mathbb{R}^{LS}$(for any consumer $i = 1, \dots, I$).[11]

Learning by i implies that k_i increases. When k_i increases, the set of contingent commodities over which i's preferences can meaningfully be defined is enlarged. Since $x_i \in X_i$ and a larger consumption set X_i may include options strictly preferred to all options in a smaller consumption set, the consumer may choose to learn in order to expand her consumption set X_i.

This observation implies, however, that if active learning is feasible, the economy generally does not satisfy the four criteria for being perfectly competitive: with no or partial spillovers, learning causes asymmetric information; with complete spillovers, i's learning affects the limits of another consumer m's consumption set, thus possibly affecting m's consumption set itself.

[11] If learning cannot be negative (forgetting is excluded), $K^0 \subseteq K_i$, so $\mathbb{R}^{Lk^0} \subseteq \mathbb{R}^{Lk_i}$ always. If learning can be negative, $\mathbb{R}^{Lk_i}$ may be smaller or larger than $\mathbb{R}^{Lk^0}$.

More specifically, the economy cannot be perfectly competitive unless either, there are no spillovers and identical learning choices are guaranteed for all consumers (which could for example, under some circumstances, result from consumers being identical); or spillovers are complete and all limits of consumption sets affected by feasible active learning are always non-binding, implying that learning cannot affect consumption sets.

Proposition 1. Assume that active learning is feasible. Then, the following holds: a) The economy is not in general perfectly competitive. b) The economy cannot be perfectly competitive unless either i) there are no spillovers, and restrictions are imposed ensuring $\Phi_i^* = \Phi_m^*$ always for all $i, m = 1, \dots, I$; or ii) spillovers are complete, and for each $i = 1, \dots, I$, $X_i \subset \mathbb{R}^{Lk_i}$ always.

Proof:

a) By assumption, active learning ($\Phi_i \neq \emptyset$) is feasible. Consider first the case with no spillovers. Let Φ_i^* denote i's active learning in equilibrium. Since in general $\Phi_i^* \neq \Phi_m^*$ for $i \neq m$, we also generally have $K_i = K^0 \cup \Phi_i^* \neq K^0 \cup \Phi_m^* = K_m$, and there is asymmetric information. Consider then the case with complete spillovers. Then, $K_i = K^0 \cup \Lambda_i \cup \Phi_i^* = K^0 \cup \Lambda_m \cup \Phi_m^* = K_m$, so there is no asymmetric information. However, since Φ_i^* spills over to m, thus affecting K_m, while m's consumption set is restricted by K_m (since $X_m \subseteq \mathbb{R}^{Lk_m}$), i's active learning Φ_i^* may affect m's consumption set X_m. According to the definition presented in Section 2, this represents an externality. With partial spillovers, both asymmetric information and externalities are present.

b) i) In the case of no spillovers, if restrictions are imposed ensuring that $\Phi_i^* = \Phi_m^*$ always for all $i, m = 1, \dots, I$, this guarantees that $K_i = K^0 \cup \Phi_i^* = K^0 \cup \Phi_m^* = K_m$ for all $i, m = 1, \dots, I$, so information is kept symmetric. The only other way to ensure symmetric information in the absence of learning spillovers would be if nature always provided the learnt information to those who did not learn it themselves, i.e., if $\Lambda_1, \dots, \Lambda_I$ is provided by nature such that $K_i = K^0 \cup \Lambda_i \cup \Phi_i^* = K^0 \cup \Lambda_m \cup \Phi_m^* = K_m$ always for all $i, m = 1, \dots, I$. This would, however, correspond to complete spillovers. ii) In the proof of Proposition 1a), external effects arise in the complete spillover case because $X_m \subseteq \mathbb{R}^{Lk_m} \subseteq \mathbb{R}^{LS}$ and because Φ_i^* affects K_m and thus k_m (where $i \neq m$), possibly affecting m's consumption set X_m itself. It is conceivable, however, that $X_i \subset \mathbb{R}^{Lk_i}$ always for all $i = 1, \dots, I$, i.e., the restriction $X_i \subseteq \mathbb{R}^{Lk_i}$ is never binding, implying that the expansion of the relevant set of commodities due to learning cannot in itself cause externalities.■

5. Other reasons for welfare relevance

If the conditions listed in Proposition 1 b ii) above hold, endogenous learning with complete or partial spillovers does not cause externalities through its impact on the relevant set of contingent commodities. However, if learning is welfare relevant for other reasons, externalities arise nevertheless.

Consider first the case where consumers have preferences over learning as such: consumers may simply enjoy acquiring or possessing new information.

If so, i's learning $\Lambda_i \cup \Phi_i$ must necessarily be an element in the consumption set X_i over which the preference relation $\gtrsim_i$ is defined. If the economy is perfectly competitive, the requirement of market completeness implies that consumers can purchase information signals $\varphi^{s',s''}$ at the market price, representing active learning. However, it is then relatively straightforward to show the following.

Proposition 2. Assume that active learning is feasible, and that no restrictions are imposed ensuring that $\Phi_i^* = \Phi_m^*$ always for every $i, m = 1, \dots, I$. Then, if any consumer $i = 1, \dots, I$ may have preferences over i's own learning $\Lambda_i \cup \Phi_i$, the economy cannot be perfectly competitive.

Proof.

Consider first the case with no spillovers. Since active learning is feasible and no restriction guarantees identical learning choices, we may have $\Phi_i^* \neq \Phi_m^*$ for some $i, m = 1, \dots, I$, yielding asymmetric information in equilibrium: $K_i = K^0 \cup \Phi_i^* \neq K^0 \cup \Phi_m^* = K_m$ for some i, m. Consider then the case with complete spillovers, i.e., $K_i = K^0 \cup \Lambda_i \cup \Phi_i^* = K^0 \cup \Lambda_m \cup \Phi_m^* = K_m$ for all $i, m = 1, \dots, I$. For a consumer i who has preferences over learning, $\Lambda_i \cup \Phi_i$ must itself be an element in the consumption set X_i of consumer i over which i's preferences are defined. However, since $\Lambda_m \cup \Phi_m^* = \Lambda_i \cup \Phi_i^*$ for all $i, m = 1, \dots, I$ due to complete spillovers, while Φ_i^* is chosen by i, X_m is directly affected by the actions of another consumer $i \neq m$, representing an externality. With partial spillovers, both asymmetric information and external effects are present. ■

Consider next the possibility that learning may affect the production technologies of firms. Assume now that consumers choose their maximally preferred bundles of commodities given prices, preferences and budget sets, taking into account any effect their learning may have on the production technologies in firms they themselves own.

Proposition 3. Assume that active learning is feasible, and that no restrictions are imposed ensuring that $\Phi_i^* = \Phi_m^*$ always for every $i, m = 1, \ldots, I$. If the production technology Y_j of any firm $j = 1, \ldots, J$ may be endogenously affected by the learning $\Lambda_i \cup \Phi_i$ of a consumer i for whom $\theta_{ij} \neq 0$, the economy cannot be perfectly competitive.

Proof.

If $0 < \theta_{ij} < 1$, i's active learning Φ_i^* may affect other consumers m for whom $0 < \theta_{mj} < 1$, i.e., other owners of the same firm, and it follows directly from the definition that there are external effects.

Consider now the case where $\theta_{ij} = 1$. Regard first the case of no spillovers. Since by assumption, no restrictions ensure $\Phi_i^* = \Phi_m^*$ always for every $i, m = 1, \ldots, I$, we can have $\Phi_i^* \neq \Phi_m^*$, yielding asymmetric information: $K_i = K^0 \cup \Phi_i^* \neq K^0 \cup \Phi_m^* = K_m$ for some $i, m = 1, \ldots, I$. Consider now the case of complete spillovers. Φ_i^* may then affect any other consumer m's knowledge K_m via the relationship $\Lambda_i \cup \Phi_i^* = \Lambda_m \cup \Phi_m^*$ for every $i, m = 1, \ldots, I$. By assumption, Φ_i can affect Y_j, the production technology of one's own exclusively owned firm. Similarly, for other consumers $m \neq i$ who are owners of other firms, i.e., $\theta_{mk} > 0$ for some $k \neq j$, m's learning $\Lambda_m \cup \Phi_m$ may affect the production technology Y_k of firm k. Thus, since $\Lambda_i \cup \Phi_i^* = \Lambda_m \cup \Phi_m^*$, i's active learning generally affects m's learning, in turn possibly affecting production technology Y_k; hence there are external effects according to the definition. With partial spillovers, both asymmetric information and external effects are generally present. ■

If active learning can be welfare relevant by influencing other economically relevant variables such as profit shares, initial endowments or preferences, similar results obtain.

6. An impossibility result

The above arguments can be summarized as follows.

Proposition 4. Assume that welfare relevant active learning is feasible, and that no restrictions are imposed ensuring $\Phi_i^* = \Phi_m^*$ always for all $i, m = 1, \dots, I$. Then, the economy cannot be perfectly competitive.

Proof.

Consider first the case of no spillovers. Since welfare relevant active learning is feasible and no restrictions ensure $\Phi_i^* = \Phi_m^*$ always for every $i, m = 1, \dots, I$, we can have $\Phi_i^* \neq \Phi_m^*$, yielding asymmetric information: $K_i = K^0 \cup \Phi_i^* \neq K^0 \cup \Phi_m^* = K_m$ for some $i, m = 1, \dots, I$.

Consider then the case of complete spillovers. Then, since $K_i = K^0 \cup \Lambda_i \cup \Phi_i^* = K^0 \cup \Lambda_m \cup \Phi_m^* = K_m$, there is no asymmetric information, but Φ_i^* may affect K_m for every other consumer m. Since learning can be welfare relevant, it can be the case that for some $i, m = 1, \dots, I$ such that $m \neq i$, and for some $j = 1, \dots, J$ such that $\theta_{ij} \neq 1$, consumer i's active learning Φ_i^* affects X_m and/or $\succsim_m$ and/or ω_m and/or θ_{mj} and/or Y_j, which represent externalities according to the definition. ■

7. Discussion

Several remarks may be in order at this point.

First, the reader may wonder why the externality problem occurring in the spillovers case cannot simply be solved through bargaining between consumers (Coase 1960). However, when information is symmetric and participation in the bargaining is voluntary, Ellingsen and Paltseva (2016) show that the Coase theorem holds in general only when $I \leq 2$.[12] An economy with only two consumers, who make their learning decisions through bilateral bargaining, would hardly satisfy the condition that all agents are price takers.

Second, note that even if there is no actual learning at all, i.e., if $\Phi_i^* = \Phi_m^* = \emptyset$ for all $i, m = 1, \dots, I$, the mere feasibility of active, welfare relevant learning still involve externalities if there are learning spillovers, possibly causing the market equilibrium to be Pareto inefficient. In particular, if active learning is costly, all welfare effects of learning are positive, and spillovers are partial or complete, a classical free-rider problem emerges: the incentives for active learning

[12] Myerson and Satterthwaite (1983) demonstrate the general impossibility of ex post efficient, non-subsidized mechanisms for bargaining between a buyer and a seller about a single object, but assume that valuations are private information. In the full spillovers case studied above, all information is symmetric.

are insufficient to achieve the Pareto optimal level of shared learning in equilibrium, since consumer i has no incentive to take into account m's welfare gain caused by i's learning ($m \neq i$). Thus, the Pareto efficient level of shared learning may well be non-empty while $\Phi_i^* = \Phi_m^* = \emptyset$ for all $i, m = 1, \dots, I$.[13] The Appendix provides a simple example.

Third, if there exists some type of information that is never relevant to anyone else but the learner, active learning of such information might not involve efficiency problems: with no spillovers, the asymmetric information might not matter; with complete spillovers, the information would not cause external effects. However, it is not easy to see what type of information this might be – particularly in a perfectly competitive economy, where the assumption of no missing markets essentially means that everything of interest is subject to trade. It is well known, for example, that asymmetric information about private preferences is often associated with inefficiency (e.g., Myerson and Satterthwaite 1983).

Fourth, note that if all new information were imposed exogenously by nature and transmitted to all consumers simultaneously, such passive learning would cause neither asymmetric information nor externalities. While the effects may be economically relevant, they would simply amount to exogenous shocks; the impact on one consumer or firm would not be caused by the actions of another economic agent, and would hence not represent externalities.

Finally, the proofs to the above propositions do not distinguish between direct spillovers (through, for example, mind-reading) and indirect spillovers dispersed through market prices. In purely speculative markets where traders have identical priors and rational expectations, it has been shown that new private information can be fully reflected in the new equilibrium prices, triggering no new trade (Tirole 1982; Milgrom and Stokey 1982). This begs the question of whether the complete and partial spillovers parts of the above proofs are valid for indirect information spillovers via markets. Effects mediated via market prices are usually not considered externalities; moreover, if new information triggers no trade, it is less obvious that such spillovers would have effects at all.

Learning in the pure speculation market models of Tirole (1982) and Milgrom and Stokey (1982), however, is not necessarily active; moreover, it is not welfare relevant. In these models, trading is a zero-sum game: learning provides no aggregate gains, and new information is useful only by providing an informational advantage vis-a-vis others. Welfare relevance, as this

[13] This is related to the reasoning of Grossman and Stiglitz (1976, 1980): If costly active information acquisition is perfectly transmitted to all economic agents, no-one has sufficient incentives to acquire the information.

concept was defined above, means that the new information could feasibly affect someone's consumption set, preferences, production technologies, initial endowments, or profit shares, even if the information were symmetrically available to all.

It is far from obvious that the results of Tirole (1982) and Milgrom and Stokey (1982) – full information dispersion, no new trade – could be replicated for active, welfare relevant learning, since this would affect rational traders' inferences about others' motives for trade. Nevertheless, note that new welfare relevant information can cause economically relevant real effects even in the absence of new trade. Consider, for example, a factory owner who learns that a costless, previously unrecognized adjustment of her factory equipment increases productivity. If this insight were fully transmitted to everyone else through markets, her learning might also improve the productivity of other firms using similar equipment, owned by other consumers – even if no new trade occured. Similarly, if consumers enjoy learning, information shared through markets would yield benefits even in the absence of new trade.

Next, if caused by dispersion of welfare relevant active learning through market prices, could such effects be considered externalities? Mas-Colell et al. (1995, p. 352) define an externality to be present "whenever the well-being of a consumer or the production possibilities of a firm are directly affected by the actions of another agent in the economy", elaborating further: "When we say "directly", we mean to exclude any effects that are mediated by prices. That is, an externality is present if, for example, a fishery's productivity is affected by the presence of a nearby oil refinery, but not simply because the fishery's profitability is affected by the price of oil (which, in turn, is in some degree affected by the oil refinery's output of oil)" (Mas-Colell et al. 1995, p. 352). One interpretation of this is that effects of information dispersed through market prices, welfare relevant or not, can be considered externalities.

However, if welfare relevant information were in fact dispersed through markets, the fundamental free-rider problem associated with learning spillovers would still be present: the information itself would be distributed for free, giving consumers insufficient incentives to take welfare effects on others into account when deciding what and how much to learn. The firm owner learning how to increase firm productivity, for example, would influence other firm owners not just through changes in the equilibrium prices per se (as when a fishery's profitability is affected by the prices of oil), but also directly, by enabling other consumers to increase the productivity of their firms.

8. Conclusions

Actively chosen welfare relevant learning generally involves market failure. If, starting from a situation of symmetric information, a consumer learns something new, information becomes asymmetric unless the new information immediately spills over to everyone else. With complete spillovers, on the other hand, chosen welfare relevant learning gives rise to externalities: choosing to learn would then essentially amount to private provision of a public good, i.e., knowledge – leaving consumers with insufficient incentives to take into account the impacts of their learning on others.

Hence, if welfare relevant active learning is feasible, the economy is generally not perfectly competitive. In fact, such economies *cannot* be perfectly competitive unless one imposes restrictions guaranteeing identical learning choices by all consumers.

The standard conditions for the first and second fundamental theorems of welfare economics, thus, cannot hold when consumers may choose to learn different welfare relevant information.

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Appendix: a simple model with active, welfare relevant learning and complete spillovers

There are I consumers. Assume complete learning spillovers. Let knowledge K be a public good, and let there be some resource R that can be spent either on consumption or learning. For every $i = 1, \ldots, I$, let the consumer's utility i be given by

(A1) $U^i = u^i(c^i, K)$

where c^i is i's consumption. Utility is quasiconcave and strictly increasing in each variable ($u_c^i > 0$, $u_K^i > 0$, subscripts denote derivatives). Furthermore, let knowledge be determined by exogenous initial knowledge K_0 plus the sum of each individual i's active learning h^i:

(A2) $K = K_0 + \sum_1^I h^i$

Let each individual i take the active learning of other consumers $m \neq i$ as exogenously given, making the market equilibrium a Nash equilbrium. With complete spillovers, there is no need for several different consumers to actively observe the same signal; however, disregard the possibility that resources are spent on wasted learning, since this obviously can occur neither in a Pareto optimum nor in a Nash equilibrium (spending resources to actively learn information provided by others cannot be a best response).

Let R^i, the total amount of resources available to consumer i, be exogenously fixed. Total resources are shared between consumption and learning. The consumption good is the numeraire, while active learning units are normalized to correspond to a unit price of 1:

(A3) $\sum_1^I R^i = \sum_1^I (c^i + h^i)$.

Finally, each consumer's budget restriction is given by

(A4) $R^i = c^i + h^i$.

The Pareto optimal supply of knowledge can be found by maximizing U^i subject to $U^m = U_0^m$ for every $m \neq i$, where U_0^m is exogenously fixed, using eqs. (A1) - (A3).

The Lagrangian is

$\mathcal{L} = u^i\left(c^i, K_0 + \sum_{m=1}^I h^m\right) - \lambda_1\left(\sum_{m=1}^I (c^m + h^m) - \sum_{m=1}^I (R^m)\right) - \sum_{m\neq i} \lambda_2^m \left(u^m\left(c^m, K_0 + \sum_{m=1}^I h^m\right) - U_0^m\right)$.

The first order condition for an interior Pareto optimal allocation is

(A5) $\sum_1^I(\frac{u_K^i}{u_c^i}) = 1,$

corresponding to the standard Samuelsonian condition for Pareto optimal supply of a public good.

In the Nash equilibrium, every individual i maximizes her own utility as given by eq. (A1), taking others' knowledge supply and her budget as given (eqs. A2 and A4). The Lagrangian for this problem is $\mathcal{L}^i = u^i\big(c^i, K_0 + \sum_1^I h^i\big) - \mu^i\big(c^i + k^i - R^i\big)$. Thus, the first order condition for an interior optimum is $u_c'^1 = u_K'^1$, or $\frac{u_K^i}{u_c^i} = 1$. By symmetry, a similar condition holds for consumer $m \neq i$, such that the market equilibrium is characterized by

(A6) $\frac{u_K^i}{u_c^i} = \frac{u_K^m}{u_c^m} = 1$ for all $i, m = 1, \dots, I,$

which is inconsistent with eq. (A5), since all marginal derivatives are strictly positive. Thus, in this economy, an interior market equilibrium cannot be Pareto efficient.

It remains to be shown that the market equilibrium may be Pareto inefficient even in the corner solution where no learning takes place. Consider the case where $\frac{u_K^i(R^i,K_0)}{u_c^i(R^i,K_0)} < 1$ for each i; that is, the marginal individual benefit of information is too small for anyone to bother to engage in learning even when all resources are spent on private consumption and no-one engages in active learning. The Nash equilibrium is then the corner solution $h^i = h^m = 0$ for every $i = 1, \dots, I$. Nevertheless, even if $\frac{u_K^i(R^i,K_0)}{u_c^i(R^i,K_0)} < 1$ for each i, we may still have $\sum_1^I(\frac{u_K^i(R^i,K_0)}{u_c^i(R^i,K_0)}) > 1$. If so, the market solution $h^i = 0$ for all $i = 1, \dots, I$ is not Pareto optimal.